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Onset of convection in a hybrid nanofluid porous layer

ABSTRACT

Thermal instability in nanofluid–porous systems is of significant interest due to its direct relevance in advanced thermal management, energy storage, and heat transfer enhancement technologies. Although extensive studies exist on convection in conventional nanofluids, the stability characteristics of hybrid nanofluids saturated in porous media—particularly under combined solutal and nanoparticle effects—remain insufficiently explored in the literature. The present study aims to investigate the onset of thermal convection in a hybrid nanofluid layer heated from below while accounting for the influence of a saturated porous medium. A linear stability framework is employed, and the governing equations are analysed using the normal mode technique to determine the conditions for stability and instability. The results reveal that the incorporation of two different nanoparticle types in the base fluid significantly delays stationary convection compared with single-nanoparticle suspensions, thereby enhancing thermal stability under top-heavy particle distributions. Furthermore, the presence of solute gradients is found to promote earlier convection onset in the heated layer. These findings provide new insights into controlling convection in hybrid nanofluid–porous systems for next-generation thermal applications.
Keywords: *Hybrid nanofluid layer; Brownian motion; Porous medium; Stationary convection; Oscillatory convection*

1. INTRODUCTION

The act of nanofluidmainlyinfluence by the physical and thermal behaviour of nano-metal-particles and the foundationliquid.Two types of nanoparticles suspended in the base fluid named as hybrid nanofluid. These quality of nanofluidsmake it more useful in various fields like residential, commercial, transportation, and industrial divisions etc. Choi and Eastman [1]first to present a class of fluidsin which the combination of nano-sized metal particlescould affect the fluid’s ability to transfer conventionalheat. Lee et al.[2]exhibit that blend of nanometre size particles and base fluid have higher thermal conductivities than the foundation fluid. Tzou [3,4] assumed the heat transport in a below heated layer of nanofluid and used the Buongiorno [5] transport equations and observed that the nanoparticles make the flow more unstable due to the existence of thermophoresis and Brownian motion.

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In the literature, various authors considered the effect of a various range of parameters on the nanofluid layer. Thetemperature-based instability in a fluid layer containing nanoparticles is discussed in detail by NieldandKuznetsov[6]. Singh et al. [7] analytically and numerically studied the salt particle drift in a Maxwell based nanofluid layer in porous media.

Kumar and Shivakumara[8] used the LTNE model to study the stability behavior of the Oldroyd B fluid layer with a constant pressure gradient and solute gradient along with heat has a significant impact on the fluid density and consequently, this coupled phenomenon can construct interesting flow behaviour, even in the fluid with uniform density distribution. Nield and Kuznetsov[9] has done the analysis of heat transport in abelow heated layer of nano liquid with concentration. The Rayleigh-Benard instability in a layer of nanofluid is considered by Dhananjay et al.[10].Sheu [11] studied the instability in porous medium and describe the effect of various parameters.Umavathi et al.[12] studied the cross-diffusion in addition to the in a nanofluid permeable layer. Sheu [13] has done the linear stability analysis of heat transport in a below heated layer of viscoelastic nanofluid and found that the region of oscillatory mode is affected by the parameters. Sharma and Gupta [14] considered theheat and solute transport in a below heated porous layer of rotating nanofluid. Pranesh et al. [15] considered the convection in water when three diffusing elements are added to the water.Mehryana et al.[16] studied the heat transport in a below heated layer of porous Cu–Al2O3/water composite nanofluidwhile the effect of magnetic field on the above problem withhalf-sinusoidal non-uniform heating is analyzed by Biswaset al.[17]. Kumar and Awasthi [18]analyzed the onset of heattransport and two solute gradients in a below heated layer ofnanofluid. Also, Kumar and Awasthi[19]analyzed the heattransport in a below heated layer of hybrid nanofluidandobserved the behaviour of various parameters.Srivastava et al.[20]examinedcompositenanofluid layer using LTNE effect and analysed consequences of top heavy and bottom-heavy nanoparticles and found that at bottom heavy arrangement is more stable than top heavyarrangement.Mehryan et al. [21] studied natural convection of magnetic hybrid nanofluid inside a double-porous medium and the results were obtained in a wide range of governing parameters such as magnetic field, viscosity parameter,porosity ratio etc.

Pundir et al. [22] examined linear stability of a fluid layer containing nanoparticles in the presence of a solute gradient. Using hybridenanofluid.Sahoo and Kumar [23] investigated the various mixtures of nanoparticles in order to find the maximum heat transfer rate such as Al_2O_3–CuO–TiO_2 ternary hybrid nanofluid were examined.Ghalam-baz et al. [24] have studied the conjugate free convection Ag–MgO in a porous enclosure using LTE analysis. The out puts show that the hybrid nanoparticles decline the heat transfer rate and flow strength.Hansda et al.[25] studied ternary hybridenanofluid in a partially heated wavy porous case for to understand the behaviour double diffusive.The thermo-physical properties of hybrid nanofluids were investigated by Huminic et al. [26] and found that the hybrid nanofluids are working fluids which could improve significantly the heat transfer in heat exchangers. Ahuja et al. [27] considered the problem to investigate the thermal convection of porous nanofluid adding rotation and magnetic field to check stability of metallic and semiconductors fluids. Manaa et al.[28] studied the problem of enhancement of micropolarhybrid nanofluid in comparison with simple nanofluid and observed that hybrid micropolarnanofluid displays more heat and mass transfer rates for thermal buoyancy when compared with traditional nanofluid.Kumar et al. [29] studied the instability of tri-hybrid nanofluid with different gravity modulation also Bhadauriaet al. [30] examined the ternary hybrid nanofluid in hele-show cell as well as throughflow which shows hele-show cell has destabilizing effect in minimizing width cell.

Manjunathan et al. [31] explored the coupled effects of thermal radiation, magnetic field strength, and buoyancy-driven nanofluid convection in a rotating configuration with an imposed heat source. Gangadharaiah et al. [32] focused on the role of internal heat generation in triggering penetrative convection within a horizontally heated composite nanofluid layer, where two distinct nanoparticle types are dispersed in a base fluid. Their study provides insight into how variations in thermal and physical parameters govern the onset of instability. Extending these investigations, Manjunathan et al. [33] addressed the problem of three-component Marangoni convection in an infinitely extended composite system, incorporating uniform heat sources or sinks in both the fluid and porous regions under a constant vertical magnetic field. In their formulation, the upper fluid boundary may be free and either adiabatic or isothermal, while the lower porous boundary is rigid and adiabatic. Further, Manjunathan et al. [34] analysed double-component magneto–Marangoni convection in a composite layer subjected to steady heat generation or absorption, bounded by adiabatic surfaces and influenced by imposed temperature gradients. Finally, Manjunathan et al. [35] presented a closed-form study of non-Darcy three-component Marangoni convection in a two-layer porous–fluid system, considering constant heat source/sink effects along with a uniform vertical magnetic field.

Most of the existing studies on thermal convection instability have focused either on the onset of convection in ordinary nanofluids within saturated porous layers or on the stability behavior of hybrid nanofluids in clear (non-porous) fluid domains, without accounting for the combined influence of porous medium effects. As a result, the convection characteristics of a hybrid nanofluid-saturated porous layer heated from below remain insufficiently explored in the current literature. This gap motivates the present work, which aims to investigate the onset of convection in such a configuration using the linear stability theory and the normal mode approach. By simultaneously incorporating hybrid nanoparticle suspension and porous medium saturation, the study provides new insights into the modification of critical conditions for convection and enhances the understanding of hybrid nanofluidbehavior in porous environments. The findings of the present study on the onset of convection in a hybrid nanofluid-saturated porous layer have important implications for several industrial and technological applications where enhanced heat transfer and thermal stability are crucial. In particular, the results can be useful in the design and optimization of porous heat exchangers, thermal insulation systems, geothermal energy extraction, electronic cooling devices, solar thermal collectors, and energy storage systems, where hybrid nanofluids are increasingly employed to improve thermal performance. Moreover, understanding the critical conditions for convection in porous environments provides guidance for controlling heat transport in advanced cooling technologies, chemical processing, and materials engineering applications involving porous structures and nanoparticle-enhanced working fluids.

2. MATHEMATICAL DESCRIPTION OF THE PROBLEM

An infinite horizontal hybrid nanofluid layer of a porous medium of thickness d of is considered between the perfectly insulating planes situated at $z^*=0$ and $z^*=d$. The Oberbeck-Boussinq approximation is employed to linearize the equations and darcy's law is assumed to hold. The temperature at the plane $z^*=d$ is assumed to be T_0^*; meanwhile the temperature at $z^*=0$ is $T_0^*+\Delta T^*$ with $\Delta T^* \ll T_0^*$. In this analysis, the dimensional variables are represented by adding asterisks; meanwhile the non-dimensional variables are without asterisks**.** Here, z-axis is taken as vertically upward and therefore, the gravitational force will be $-g\hat{\boldsymbol{k}}$. The changes in temperature and concentration are assumed to be very small. The hybrid nanofluid mixture is considered to be homogeneous and will be in local thermal equilibrium state.

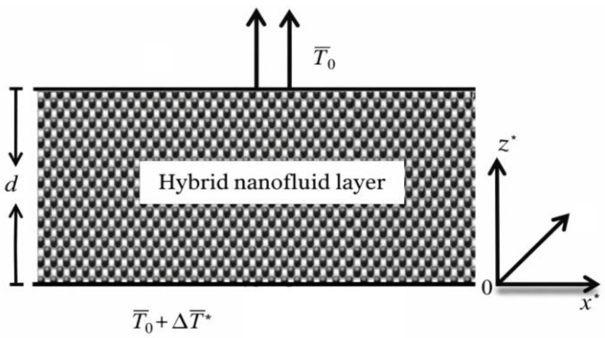

Figure 1. Schematic illustration of the proposed problem

(i) **Incompressible, Newtonian, and laminar flow**:

The hybrid nanofluid is assumed incompressible and Newtonian because the analysis focuses on the onset of convection, where flow velocities are very small. Near the critical instability threshold, the motion remains laminar, making this assumption appropriate.

(ii) **Stable nanoparticle suspension**:

The nanoparticles are assumed to remain uniformly dispersed due to stabilization techniques such as surfactants or surface charge methods. This prevents agglomeration and allows the hybrid nanofluid to be modeled as a single-phase continuum.

(iii) **No chemical reactions and negligible inter-particle forces**:

Chemical reactions are neglected since the study concerns purely thermal convection. Furthermore, interaction forces between nanoparticle species are ignored because the suspension is assumed dilute and stable, consistent with commonly adopted hybrid nanofluid models.

(iv) **Thermal equilibrium between phases**:

The nanoparticle phases and base fluid are assumed to be in local thermal equilibrium. This is justified because nanoparticles are extremely small and thermal exchange between particles and fluid occurs rapidly, supporting the single-temperature approximation.

(v) **Boussinesq approximation and constant properties**:

Density variation is neglected except in the buoyancy term, following the Boussinesq hypothesis, which is valid for small temperature gradients. Other thermophysical properties are assumed constant to simplify the linear stability framework and highlight the dominant convection mechanism.

(vi) **Spherical nanoparticles and negligible radiative effects**:

Spherical nanoparticles are considered for mathematical simplicity and because many experimental hybrid nanofluids use nearly spherical particles. Particle–particle interactions are neglected due to electrostatic stabilization, and radiative heat transfer is ignored as its contribution is exceptionally small compared to conduction and convection in the present configuration.

The principal equations of conservation which characterizing the flow of composite nano-fluid layer in dimensional form according to works cited in the references (Tzou [3,4], Buongiorno [5]) are as follows:

Conservation of mass;

$$\nabla\cdot\boldsymbol{u}^*=0, \qquad (1)$$

Conservation of momentum;

$$\rho\left(\frac{\partial\boldsymbol{u}^*}{\partial t^*}+(\boldsymbol{u}^*\cdot\nabla^*)\boldsymbol{u}^*\right)=-\nabla^*p^*-\rho g\mathbf{k}-\frac{\mu}{K}\nabla^{*2}\boldsymbol{u}^*, \qquad (2)$$

Conservation of energy;

$$(\rho c)_m \frac{\partial T^*}{\partial t^*} + (\rho c)_f\, (\boldsymbol{u}^* \cdot \nabla^*) T^* = \varepsilon (\rho c)_{p_1} \left[D_{B_1} \nabla^* \phi_1^* \cdot \nabla^* T^* + \left(D_{T_1} / T_0^* \right) \nabla^* T^* \cdot \nabla^* T^* \right]$$
$$+\varepsilon (\rho c)_{p_2} \left[D_{B_2} \nabla^* \phi_2^* \cdot \nabla^* T^* + \left(D_{T_2} / T_0^* \right) \nabla^* T^* \cdot \nabla^* T^* \right] + \kappa_m \nabla^{*2} T^*, \quad (3)$$

Conservation of nanoparticles;

$$\frac{\partial \phi_1^*}{\partial t^*} + \frac{1}{\varepsilon} (\boldsymbol{u}^* \cdot \nabla^*) \phi_1^* = D_{B_1} \nabla^{*2} \phi_1^* + \left(D_{T_1} / T_0^* \right) \nabla^{*2} T^* . \quad (4)$$

$$\frac{\partial \phi_2^*}{\partial t^*} + \frac{1}{\varepsilon} (\boldsymbol{u}^* \cdot \nabla^*) \phi_2^* = D_{B_2} \nabla^{*2} \phi_1^* + \left(D_{T_2} / T_0^* \right) \nabla^{*2} T^* . \quad (5)$$

In the Boussinesq approximation, the density equation is

$$\rho_f = \rho_{f_0} \left[1 - \beta_T (T^* - T_0^*) \right] . \quad (6)$$

Also

$$\rho = \phi_1^* \rho_{p_1} + \phi_2^* \rho_{p_2} + (1 - \phi_1^* - \phi_2^*) \rho_f . \quad (7)$$

Here ρ_{p_1}, ρ_{p_2} and ρ_f are the nano-particles mass densities and base fluid density while ρ and ρ_{f_0} are nanofluid density and base fluid density at reference temperature T_0. Using equations (6) and (7), we obtain

$$\rho = \phi_1^* \rho_{p_1} + \phi_2^* \rho_{p_2} + (1 - \phi_1^* - \phi_2^*) \rho_{f_0} \left[1 - \beta_T (T^* - T_0^*) \right] . \quad (8)$$

Since the volumetric fraction of nanoparticles is only a few percent (Choi and Eastman [1]) so taking the density of base fluid as that of the density of nanofluid i.e. $\rho \cong \rho_{f_0}$ as adopted by (Tzou[3]), the specific weight becomes

$$\rho g \cong \phi_1^* \rho_{p_1} + \phi_2^* \rho_{p_2} + (1 - \phi_1^* - \phi_2^*) \rho \left[1 - \beta_T (T^* - T_0^*) \right] . \quad (9)$$

Here, the nanofluid Darcy velocity is taken as $\boldsymbol{u}^*(m/s) = (u^*, v^*, w^*)$; $t^*(s)$ is the time; $\mathbf{g}$ is the gravitational acceleration; $T^*(^0K)$ is the nanofluid temperature; $\beta_T(^0K^{-1})$ is the thermal volumetric expansion coefficient; ϕ_1^*, ϕ_2^* are the nano-particle volume fractions; $\mu(N\,s/m^2)$ is the viscosity; K the medium permeability; $D_{B_1}, D_{B_2}(m^2/s)$ are the Brownian diffusion coefficients; $D_{T_1}, D_{T_2}(m^2/s)$ are the thermophoresis diffusion coefficients; $D_S(m^2/s)$ is the solutal diffusivity; $\kappa_m(W/mK)\,(=\varepsilon\kappa$, where the thermal conductivity of the fluid) is the effective thermal conductivity of the porous medium; c is the specific heat of nanofluid at constant pressure; $(\rho c)_m$ is the effective heat capacity of the medium; $(\rho c)_f$ is the effective heat capacity of the fluid; $(\rho c)_p$ is the effective heat capacity of the material constituting nanoparticles; ε is the porosity of the porous medium. Also, we have assumed that the temperature and nano-particles volume fractions are fixed on both the boundaries. The conditions on the boundary are defined as;

$$\left. w^* = 0,\ \frac{\partial w^*}{\partial z^*} + \lambda_1 d \frac{\partial^2 w^*}{\partial z^{*2}} = 0,\ T^* = T_0^* + \Delta T^*,\ \phi_1^* = \phi_{10}^*,\ \phi_2^* = \phi_{20}^* \right\}\ at\ z^* = 0 \quad (10)$$

$$\left. w^* = 0,\ \frac{\partial w^*}{\partial z^*} - \lambda_2 d \frac{\partial^2 w^*}{\partial z^{*2}} = 0,\ T^* = T_0^*,\ \phi_1^* = \phi_{11}^*,\ \phi_2^* = \phi_{21}^* \right\}\ at\ z^* = d . \quad (11)$$

Here d is the dimensional layer depth while λ_1 and λ_2 are the parameters which take the value zero for the case of a rigid boundary and infinity for the case of a free boundary.

We admit that in a number of contexts, the selection of boundary conditions forced on ϕ_1^* and ϕ_2^* are somewhat subjective. It might be claimed that on the boundaries, zero particle flux is more practical, but then one is faced with the problem that it appears that no steady state solution for the basic conduction equations is then possible (we have tried to find one and met a contradiction) so that in order to find the analytical solution for considered problem it is necessary to restrict the basic profile for ϕ_1^*, ϕ_2^* and at that stage our choice of boundary conditions is seen to be quite realistic**.**

Now, we introduce the non-dimensional variables

$(x,y,z),\ (u,v,w),\ t,\ p,\phi_1,\ \phi_2$ and T is defined as

$$(x,y,z)=(x^*,y^*,z^*)/d\ ,$$
$$(u,v,w)=(u^*,v^*,w^*)d/\alpha_m\ ,$$
$$t=t^*\alpha_m/\sigma d^2\ ,\ p=p^*K/\mu\alpha_m\ ,$$
$$\phi_1=\left(\phi_1^*-\phi_{10}^*\right)/\left(\phi_{11}^*-\phi_{10}^*\right),$$
$$\phi_2=\left(\phi_2^*-\phi_{20}^*\right)/\left(\phi_{21}^*-\phi_{20}^*\right),$$
$$T=\left(T^*-T_0^*\right)/\Delta T^*,$$

where $\alpha_m=\kappa_m/\left(\rho c\right)_f\ \left(m^2/s\right)$ is the thermal diffusivity of the porous medium, $\sigma=\left(\rho c\right)_m/\left(\rho c\right)_f$ is the heat capacity ratio parameter and $\phi_{10}^*,\ \phi_{20}^*$ are the reference scale nanoparticles fractions.

. The non-dimensional form of the equations (1) to (6) can be stated that

$$\nabla\cdot\boldsymbol{u}=0\ , \qquad (12)$$

$$\frac{\eta}{Va}\frac{\partial\boldsymbol{u}}{\partial t}=-\nabla p-\boldsymbol{u}+RaT\hat{e}_z-Rn_1\phi_1\hat{e}_z-Rn_2\phi_2\hat{e}_z-Rm\hat{e}_z\ , \qquad (13)$$

$$\frac{\partial T}{\partial t}+(\boldsymbol{u}\cdot\nabla)T=\nabla^2T+\frac{N_{B_1}}{Ln_1}\nabla\phi_1.\nabla T+\frac{N_{A_1}N_{B_1}}{Ln_1}\nabla T.\nabla T+\frac{N_{B_2}}{Ln_2}\nabla\phi_2.\nabla T+\frac{N_{A_2}N_{B_2}}{Ln_2}\nabla T.\nabla T\ , \qquad (14)$$

$$\frac{1}{\sigma}\frac{\partial\phi_1}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{u}\cdot\nabla)\phi_1=\frac{1}{Ln_1}\nabla^2\phi_1+\frac{N_{A_1}}{Ln_1}\nabla^2T \qquad (15)$$

$$\frac{1}{\sigma}\frac{\partial\phi_2}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{u}\cdot\nabla)\phi_2=\frac{1}{Ln_2}\nabla^2\phi_2+\frac{N_{A_2}}{Ln_2}\nabla^2T\ . \qquad (16)$$

The dimensionless boundary conditions take the form

$$\left.w=0,\ \frac{\partial w}{\partial z}+\lambda_1 d\frac{\partial^2 w}{\partial z^2}=0,\ \phi_1=0,\ \phi_2=0,\ T=1\right\}$$
at $z=0$ *(17)*

$$\left.w=0,\ \frac{\partial w}{\partial z}-\lambda_2 d\frac{\partial^2 w}{\partial z^2}=0,\ \phi_1=1,\ \phi_2=1,\ T=0\right\}$$
at $z=1$ *(18)*

The non-dimensional parameters used above are defined as; the Prandtl number, $\mathrm{Pr}=\mu/\rho\alpha_m$; Darcy number, $Da=K/d^2$; Vadasz number, $Va=\varepsilon^2\,\mathrm{Pr}/Da$; thermo-nanofluid Lewis numbers, $Ln_1=\alpha_m/D_{B_1}$, $Ln_2=\alpha_m/D_{B_2}$; thermal Rayleigh-Darcy number,

$Ra=\rho g\beta_T Kd\Delta T^*/\mu\alpha_m$;

the basic density Rayleigh number, $Rm=\left[\rho_{p_1}\phi_{10}^*+\rho_{p_2}\phi_{20}^*+\rho\left(1-\phi_{10}^*-\phi_{20}^*\right)\right]gKd/\mu\alpha_m$; the nano-particle concentration Rayleigh numbers, $Rn_1=\left[\left(\rho_{p_1}-\rho\right)\left(\phi_{11}^*-\phi_{10}^*\right)\right]gKd/\mu\alpha_m$ and $Rn_2=\left[\left(\rho_{p_2}-\rho\right)\left(\phi_{21}^*-\phi_{20}^*\right)\right]gKd/\mu\alpha_m$; the modified diffusivity ratios,

$N_{A_1}=D_{T_1}\Delta T^*/D_{B_1}T_0^*\left(\phi_{11}^*-\phi_{10}^*\right)$ and

$N_{A_2}=D_{T_2}\Delta T^*/D_{B_2}T_0^*\left(\phi_{21}^*-\phi_{20}^*\right)$ and the modified particle-density increments $N_{B_1}=\left(\rho c\right)_{p_1}\varepsilon\left(\phi_{11}^*-\phi_{10}^*\right)/\left(\rho c\right)_f$

and $N_{B_2}=\left(\rho c\right)_{p_2}\varepsilon\left(\phi_{21}^*-\phi_{20}^*\right)/\left(\rho c\right)_f$. We neglect the terms, which are the product of ϕ_1 and ϕ_2 with T for linearization of equation (13) following the concept of Oberbeck-Boussinesq approximation since small thermal gradients in a dilute suspension of nano-particles is considered.

3. STABILITY ANALYSIS

Basic State

The basic state of composite nanofluid layer is assumed to be time independent and the given by the expressions

$$\boldsymbol{u}=(0,\ 0,\ 0),\quad p=p_b(z),$$

$$T=T_b(z),\quad \phi_1=\phi_{1b}(z),\quad \phi_2=\phi_{2b}(z). \tag{19}$$

Equations (10) to (15) take the form with the above values

$$-\frac{dp_b}{dz}+RaT_b-Rn_1\phi_{1b}-Rn_2\phi_{2b}-Rm=0, \tag{20}$$

$$\frac{d^2T_b}{dz^2}+\frac{N_{B_1}}{Ln_1}\left(\frac{d\phi_{1b}}{dz}.\frac{dT_b}{dz}\right)+\frac{N_{A_1}N_{B_1}}{Ln_1}\left(\frac{dT_b}{dz}\right)^2+ +\frac{N_{B_2}}{Ln_2}\left(\frac{d\phi_{2b}}{dz}.\frac{dT_b}{dz}\right)+\frac{N_{A_2}N_{B_2}}{Ln_2}\left(\frac{dT_b}{dz}\right)^2=0 \tag{21}$$

$$\frac{d^2\phi_{1b}}{dz^2}+N_{A_1}\frac{d^2T_b}{dz^2}=0. \tag{22}$$

$$\frac{d^2\phi_{2b}}{dz^2}+N_{A_2}\frac{d^2T_b}{dz^2}=0. \tag{23}$$

Boundary conditions become

$$\phi_{1b}(0)=0,\ \phi_{2b}(0)=0,\ T_b(0)=1,\ \phi_{1b}(1)=1,$$

$$\phi_{2b}(1)=1 \text{ and } T_b(1)=0. \tag{24}$$

In frequently cases of nano-fluid layers, $\frac{Ln_1}{\phi_{11}^*-\phi_{10}^*}$ and $\frac{Ln_2}{\phi_{21}^*-\phi_{20}^*}$ are very large and are of order 10^5-10^6 (Buongiorno[5]) and in addition the nano-particle fraction decrement $\left(\phi_{11}^*-\phi_{10}^*\right)$ and $\left(\phi_{11}^*-\phi_{10}^*\right)$ are characteristically not smaller than 10^{-3} and therefore, we get a conclusion that Ln_1 and Ln_2 are large and is of order 10^2-10^3. It can also be observed that N_{A_1} and N_{A_2} will be lesser than 10.

Using this approximation, the basic solution can be written as follows;

$$\phi_{1b}(z)=z,\ \phi_{2b}(z)=z$$

$$\text{and } T_b(z)=1-z. \tag{25}$$

Perturbed state

The small perturbations are imposed to the basic state of the composite nanofluid layer, the parameters become $\boldsymbol{u}=\boldsymbol{u}(0,0,0)+\boldsymbol{u}(u',v',w')$, $p=p_b+p'$, $T=T_b+T'$, $\phi_1=\phi_{1b}+\phi_1'$ and $\phi_2=\phi_{2b}+\phi_2'$.

Here prime quantities are the quantities in perturbed state. The linear stability is analyzed in the present study and therefore, the nonlinear terms have been neglected. The linear governing equations are

$$\nabla\cdot\boldsymbol{u}'=0, \tag{26}$$

$$\frac{\varepsilon}{\sigma Va}\frac{\partial\boldsymbol{u}'}{\partial t}=-\nabla p'-\boldsymbol{u}'+RaT'\hat{e}_z-Rn_1\phi_1'\hat{e}_z-Rn_2\phi_2'\hat{e}_z, \tag{27}$$

$$\frac{\partial T'}{\partial t}-w'=\nabla^2T'+\frac{N_{B_1}}{Ln_1}\left(\frac{\partial T'}{\partial z}-\frac{\partial\phi_1'}{\partial z}\right)-\frac{2N_{A_1}N_{B_1}}{Ln_1}\frac{\partial T'}{\partial z}+\frac{N_{B_2}}{Ln_2}\left(\frac{\partial T'}{\partial z}-\frac{\partial\phi_2'}{\partial z}\right)-\frac{2N_{A_2}N_{B_2}}{Ln_2}\frac{\partial T'}{\partial z}, \tag{28}$$

$$\frac{1}{\sigma}\frac{\partial\phi_1'}{\partial t}+\frac{1}{\varepsilon}w'=\frac{1}{Ln_1}\nabla^2\phi_1'+\frac{N_{A_1}}{Ln_1}\nabla^2T'. \tag{29}$$

$$\frac{1}{\sigma}\frac{\partial\phi_2'}{\partial t}+\frac{1}{\varepsilon}w'=\frac{1}{Ln_2}\nabla^2\phi_2'+\frac{N_{A_2}}{Ln_2}\nabla^2T'. \tag{30}$$

In non-dimensional form, corresponding to perturbation equations where the horizontal boundaries are electrically non-conducting, the resulting conditions at the boundaries are

$$w'=0,\ \frac{\partial w'}{\partial z}+\lambda_1\frac{\partial^2w'}{\partial z^2}=0,\ \phi_1'=0,\ \phi_2'=0,\ T'=0 \text{ at } z=0 \tag{31}$$

$$w'=0,\ \frac{\partial w'}{\partial z}-\lambda_2\frac{\partial^2w'}{\partial z^2}=0,\ \phi_1'=0,\ \phi_2'=0,\ T'=0 \text{ at } z=1. \tag{32}$$

The parameter Rm is only a part of the necessary static pressure gradient and is not implicated in the above equations. In case of a regular binary fluid (not a nanofluid) the parameters

Rn, N_{A_1}, N_{A_2}, N_{B_1} and N_{B_2}

will vanish and second term in L.H.S in equation (29) and (30) are missing because $d\phi_{1b}/dz=0$ and $d\phi_{2b}/dz=0$.

Eliminating p' by operating a curl twice on (27), we obtain

$$\frac{\varepsilon}{\sigma Va}\frac{\partial}{\partial t}\nabla^2 w' = -\nabla^2 w' + Ra\nabla_H^2 T' - Rn_1\nabla_H^2\phi_1' - Rn_2\nabla_H^2\phi_2' \quad (33)$$

Here ∇_H^2 is the usual two-dimensional Laplacian operator in the horizontal plane. The differential equations (28) to (30) and (33) with boundary conditions (31) and (32) constitute a linear boundary value problem that can be solved using the method of normal modes. Now analyzing the perturbations into normal modes, we assume that the perturbation quantities are of the form

$$\left[w,\, T',\, \phi_1',\, \phi_2'\right] = \left[W(z),\, \Theta(z),\, \Phi_1(z),\, \Phi_2(z)\right]\exp\{ik_x x + ik_y y + nt\}, \quad (34)$$

where k_x and k_y are the wave numbers in x and y directions respectively, $k=(k_x^2+k_y^2)^{1/2}$ is the resultant wave number of propagation and $n=\omega_r+i\omega_i$ is a complex constant which predict the growth rate of disturbances. The system is stable if $\omega_r<0$ for all modes while unstable if $\omega_r>0$ for atleast one mode.

Using equation (34) in the equations (28) to (30) and (33), we get

$$\left(\frac{n\varepsilon}{\sigma Va}+1\right)\left(D^2-k^2\right)W + Rak^2\Theta - Rn_1k^2\Phi_1 - Rn_2k^2\Phi_2 = 0 , \quad (35)$$

$$W + \left(D^2-k^2-n+\frac{N_{B_1}}{Ln_1}D-\frac{2N_{A_1}N_{B_1}}{Ln_1}D+\frac{N_{B_2}}{Ln_2}D-\frac{2N_{A_2}N_{B_2}}{Ln_2}D\right)\Theta - \frac{N_{B_1}}{Ln_1}D\Phi_1 - \frac{N_{B_2}}{Ln_2}D\Phi_2 = 0 , \quad (36)$$

$$\frac{W}{\varepsilon}-\frac{N_{A_1}}{Ln_1}\left(D^2-k^2\right)\Theta-\left(\frac{1}{Ln_1}\left(D^2-k^2\right)-\frac{n}{\sigma}\right)\Phi_1=0 , \quad (37)$$

$$\frac{W}{\varepsilon}-\frac{N_{A_2}}{Ln_2}\left(D^2-k^2\right)\Theta-\left(\frac{1}{Ln_2}\left(D^2-k^2\right)-\frac{n}{\sigma}\right)\Phi_2=0 . \quad (38)$$

The case of both free boundaries is considered for the stability analysis. Therefore, the boundary conditions are

$$W=0,\ D^2W=0,\ \Theta=0,\ \Phi_1=0,\ \Phi_2=0 \quad at\ z=0\ and\ z=1. \quad (39)$$

The solutions of the equations (35) to (38) taken as

$$W=W_0\,Sin\pi z,\ \Theta=\Theta_0\,Sin\pi z,\ \Phi_1=\Phi_{10}\,Sin\pi z\ and\ \Phi_2=\Phi_{20}\,Sin\pi z. \quad (40)$$

Substituting equation (40) in equations (35) to (38) and integrating from $z=0$ and $z=1$, we have the following matrix equation

$$\left[a_{ij}\right]\left[W_0\ \ \Theta_0\ \ \Phi_1\ \ \Phi_2\right]^T=0 . \quad (41)$$

Here,

$$a_{11}=\left(\frac{n\varepsilon}{\sigma Va}+1\right)\delta^2, a_{12}=-Ra\,k^2, a_{13}=Rn_1k^2, a_{14}=Rn_2k^2, a_{21}=1, a_{22}=-\left(\delta^2+n\right),$$

$$a_{23}=0, a_{24}=0, a_{31}=1/\varepsilon,\ a_{32}=N_{A_1}\delta^2/Ln_1,\ a_{33}=n/\sigma+\left(\delta^2/Ln_1\right), a_{34}=0,$$

$$a_{41}=1/\varepsilon, a_{42}=N_{A_2}\delta^2/Ln_2, a_{43}=0, a_{44}=n/\sigma+\left(\delta^2/Ln_2\right).$$

Also $\delta^2 = \pi^2 + k^2$ is total wave number. The nontrivial solution of the above homogeneous equations requires that

$$Ra = \frac{\delta^2\left(n+\delta^2\right)}{k^2}\left(1+\frac{n\varepsilon}{\sigma Va}\right) - \frac{\sigma\left(Ln_1\left(n+\delta^2\right)+\varepsilon\delta^2 N_{A_1}\right)}{\varepsilon(nLn_1+\sigma\delta^2)}Rn_1 - \frac{\sigma\left(Ln_2\left(n+\delta^2\right)+\varepsilon\delta^2 N_{A_2}\right)}{\varepsilon(nLn_2+\sigma\delta^2)}Rn_2, \quad (42)$$

Now, we put $n = i\omega$ in equation (44) and we obtain

$$Ra = \Delta_1 + i\omega\Delta_2. \quad (43)$$

Here

$$\Delta_1 = \frac{\delta^2}{k^2}\left(\delta^2 - \frac{\varepsilon\omega^2}{\sigma Va}\right) - \left(\frac{\delta^4\sigma^2 Ln_1 + \sigma\omega^2 Ln_1^2}{(\delta^4\varepsilon\sigma^2 + \varepsilon\omega^2 Ln_1^2)} + \frac{\delta^4\sigma^2 N_{A_1}}{(\delta^4\sigma^2+\omega^2 Ln_1^2)}\right)Rn_1$$
$$-\left(\frac{\delta^4\sigma^2 Ln_2 + \sigma\omega^2 Ln_2^2}{(\delta^4\varepsilon\sigma^2 + \varepsilon\omega^2 Ln_2^2)} + \frac{\delta^4\sigma^2 N_{A_2}}{(\delta^4\sigma^2+\omega^2 Ln_2^2)}\right)Rn_2 \quad (44)$$

$$\Delta_2 = \frac{\delta^2\left(\delta^2\varepsilon+\sigma Va\right)}{\sigma k^2 Va} + \frac{\sigma\left(\varepsilon N_{A_1} + Ln_1 - \sigma\right)Ln_1}{\varepsilon(\delta^4\sigma^2+\omega^2 Ln_1^2)}Rn_1 + \frac{\sigma\left(\varepsilon N_{A_2} + Ln_2 - \sigma\right)Ln_2}{\varepsilon(\delta^4\sigma^2+\omega^2 Ln_2^2)}Rn_2. \quad (45)$$

The Rayleigh number Ra must be real as it is a physical quantity. Hence, it can be concluded from the equation (43) that either $\omega = 0$ (exchange of stabilities, steady state) or $\Delta_2 = 0$ ($\omega \neq 0$, overstability or oscillatory onset).

Non-Oscillatory Convection

Steady onset corresponds to $\omega = 0$ and Rayleigh number is given by

$$Ra^{st} = \frac{\delta^4}{k^2} - \frac{\left(Ln_1 + \varepsilon N_{A_1}\right)}{\varepsilon}Rn_1 - \frac{\left(Ln_2 + \varepsilon N_{A_2}\right)}{\varepsilon}Rn_2, \quad (46)$$

The critical cell size at the onset of instability is obtained from the condition $\frac{\partial}{\partial k}Ra = 0$.

$$k_c = \pi, \quad (47)$$

Thus, for steady onset, the corresponding critical thermal Rayleigh number will become

$$Ra^{st} = 4\pi^2 - \frac{\left(Ln_1 + \varepsilon N_{A_1}\right)}{\varepsilon}Rn_1 - \frac{\left(Ln_2 + \varepsilon N_{A_2}\right)}{\varepsilon}Rn_2, \quad (48)$$

For $Rn_2 = 0$, equation (48) becomes

$$Ra^{st} = 4\pi^2 - \frac{\left(Ln_1 + \varepsilon N_{A_1}\right)}{\varepsilon}Rn_1. \quad (49)$$

This equation is same as the equation given by Sheu [11] Also, It should be noted that equation (46) is independent of Prandtl number.

Non-Oscillatory Convection

For oscillatory onset, $\Delta_2 = 0$ and $\omega \neq 0$, which gives the expression of the form

$$a_2(\omega^2)^2 + a_1(\omega^2) + a_0 = 0. \quad (50)$$

Here,

$$a_2 = \left(\frac{\delta^2\varepsilon}{\sigma} + Va\right)\varepsilon Ln_1^2 Ln_2^2$$

$$a_1 = \delta^4\varepsilon\sigma^2\left(\frac{\delta^2\varepsilon}{\sigma}+Va\right)\left(Ln_1^2 + Ln_2^2\right) + \left(Ln_1 + \varepsilon N_{A_1} - \sigma\right)\sigma Va\, k^2 Ln_1 Ln_2^2 Rn_1$$
$$+\left(Ln_2 + \varepsilon N_{A_2} - \sigma\right)\sigma Va\, k^2 Ln_1^2 Ln_2 Rn_2$$

$$a_0 = \delta^8\varepsilon\sigma^4\left(\frac{\delta^2\varepsilon}{\sigma}+Va\right) + \left(Ln_1 + \varepsilon N_{A_1} - \sigma\right)\sigma^3 k^2\delta^4 Va\, Ln_1 Rn_1$$
$$+\left(Ln_2 + \varepsilon N_{A_2} - \sigma\right)\sigma^3 k^2\delta^4 Va\, Ln_2 Rn_2.$$

If equation (50) does not admit any positive value of ω^2 then oscillatory instability is not possible.Thus from Eq. (43) and (44), oscillatory Rayleigh number is given by

$$Ra^{osc} = \frac{\delta^2}{k^2}\left(\delta^2 - \frac{\varepsilon\omega^2}{\sigma Va}\right) - \left(\frac{\delta^4\sigma^2 Ln_1 + \sigma\omega^2 Ln_1^{\,2} + \varepsilon\delta^4\sigma^2 N_{A_1}}{\varepsilon(\delta^4\sigma^2 + \omega^2 Ln_1^{\,2})}\right) Rn_1 - \left(\frac{\delta^4\sigma^2 Ln_2 + \sigma\omega^2 Ln_2^{\,2} + \varepsilon\delta^4\sigma^2 N_{A_2}}{\varepsilon(\delta^4\sigma^2 + \omega^2 Ln_2^{\,2})}\right) Rn_2 , \quad (51)$$

If we are able to find two positive values of ω^2 then the minimum of (51) gives the oscillatory Rayleigh number for ω^2. If there is only one positive value of ω^2 then by substituting this positive value into (51), we get the oscillatory Rayleigh number.

Case of Overstability

In this section, we check the possibility of occurrence of overstability. Since, we wish to find the Rayleigh number for the onset of instability via a state of pure oscillation. So, it suffices to find the conditions for which equation (52) will have solution with real values of ω.

The sum of the roots of equation (50) is $-(a_1/a_2)$ while the product of the roots is (a_0/a_2). Now a_2 is always positive and a_0 and a_1 are positive if $Rn_1 > 0$, $Rn_2 > 0$, $Ln_1 + \varepsilon N_{A_1} > \sigma$ and $Ln_2 + \varepsilon N_{A_2} > \sigma$.

Hence, these inequalities are sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

4. RESULTS AND DISCUSSION

In this section, we describe our results numerically. Stationary thermal Rayleigh number for composite nanofluid heated and soluted from below is given by equation (48) and the expression of the oscillatory thermal Rayleigh number is obtained analytically using equation (51) where ω^2 is given by equation (50).It is obvious that both stationary Rayleigh number and oscillatory Rayleigh number for composite nanofluid do not depend on N_{B_1} and N_{B_2} because the effect of N_{B_1} and N_{B_2} in equation (36) is cancelled due to integration of orthogonal functions.The impact of Brownian motions and thermophoresis in the thermal energy equations of instability do not appear. The Brownian motions and thermophoresis directly contribute in the equation expressing the conservation of nanoparticles to produce their effects. In this way, the temperature and nanoparticle densities are coupled in a particular way in which the instability is almost purely a phenomenon due to buoyancy coupled with the conservation of nanoparticle motions.

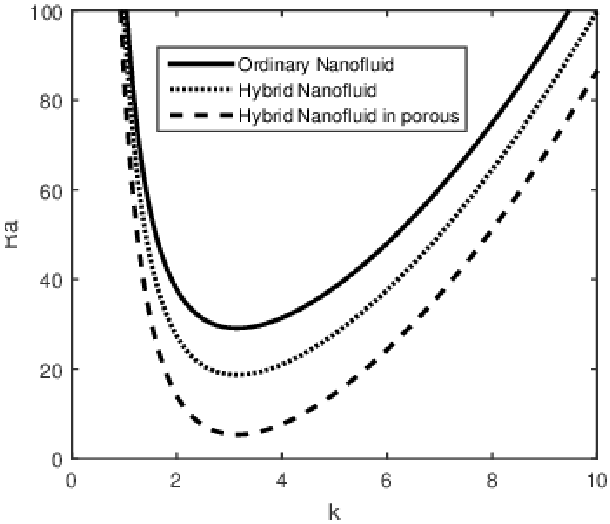

Figure 2..Comparisonof Ra for nanofluid, hybrid nanofluid with porous medium

Figure 2. shows the comparison between the Rayleigh numbers of ordinary nanofluid, composite nanofluid and porous medium with hybrid nanofluidfor $Ln_1 = 50$, $Ln_2 = 50$, $N_{A_1} = 2$, $N_{A_2} = 2$, $\varepsilon = 0.6$, $Rn_1 = 0.2$ and $Rn_2 = 0.2$. Here, positive values of Rn_1 and Rn_2 mean that a top-heavy distribution is considered. It has been observed that Rayleigh number for composite nanofluid is more than the Rayleigh number of ordinary nanofluid. This shows that mixture of two different nanoparticles more stabilizes the stationary convection as compare of the single nanoparticles in case of top-heavy distribution.

Figure 3 shows the neutral curves for different values of Thermo-nanofluid Lewis number Ln_2 with fixed values of other parameters. It has been observed that stationary Rayleigh number decreases with increase the thermo-nanofluid Lewis number.

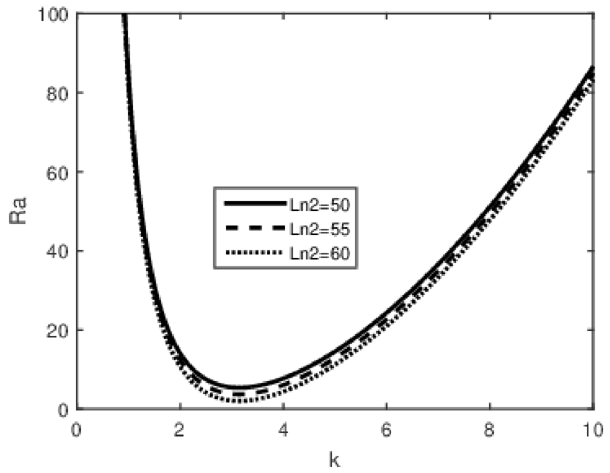

Figure 3. Variation of Ra with Thermos-nanofluid Lewise number

Thus, thermo-nanofluid Lewis number destabilizes the stationary convection in case of top-heavy distribution while it stabilizes the stationary convection in case of bottom-heavy distribution**.**

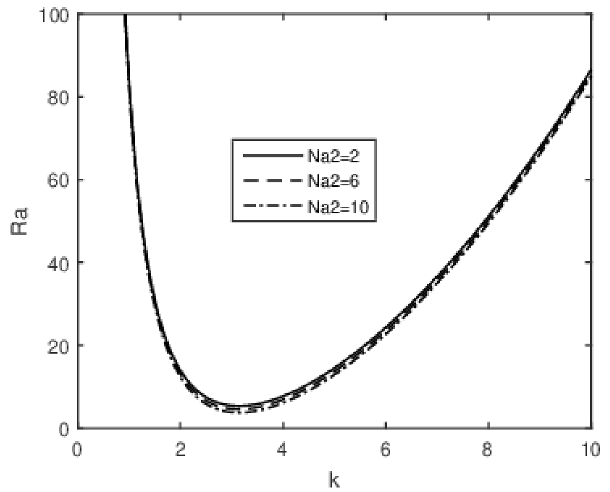

Figure 4. Variation of Ra with Modified diffusivity ratios

Figure 4 shows the neutral curves for different values of modified diffusivity ratio N_{A_2} with fixed values of other parameters. It has been observed that stationary Rayleigh number decreases with increase the first modified diffusivity ratio. Thus, first modified diffusivity ratio destabilizes the stationary convection in case of top-heavy distribution**.**

Figure 5 shows the neutral curves for different values Nanoparticle concentration Rayleigh number R_{n_2} with fixed values of other parameters. It has been observed that stationary Rayleigh number decreases with increase.

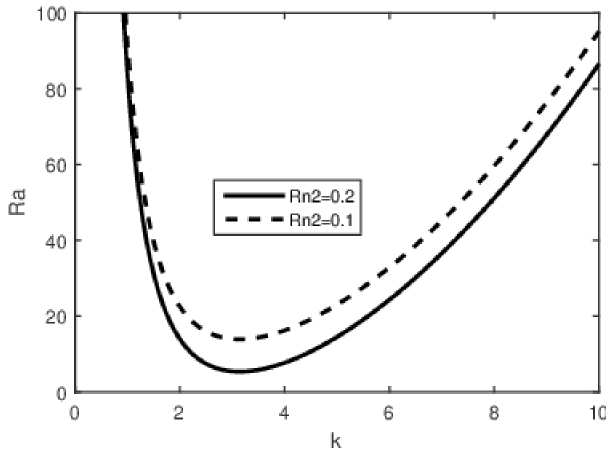

Figure 5. Variation of Ra with Nanoparticle concentration Rayleigh number

Nanoparticle concentration Rayleigh number. Thus destabilizes the stationary convection in case of top-heavy distribution**.**

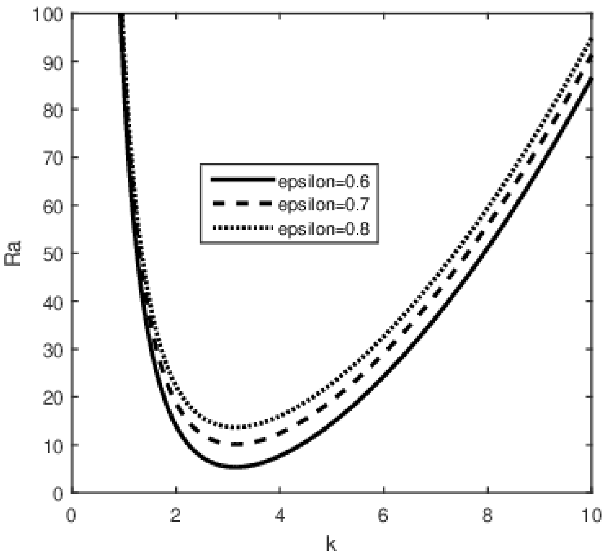

Figure 6. Variation of Ra with porosity

Figure 6 shows the Impact of porosity for different values of Ra where it is discovered the porosity destabilizes the stationary convection in case of top-heavy distribution.

5. CONCLUSIONS

The present study provides a theoretical stability characterization of thermal convection in a hybrid nanofluid-saturated porous layer heated from below, using the linear stability framework and normal mode analysis under free–free boundary conditions. Unlike earlier investigations that primarily addressed either ordinary nanofluids in porous media or hybrid nanofluids in clear fluids, this work offers a novel contribution by incorporating the combined influence of hybrid nanoparticle suspension and porous medium saturation on the critical onset of convection.

The analysis demonstrates that the initiation of instability is governed mainly by buoyancy–nanoparticle coupling, while Brownian diffusion effects do not alter the critical Rayleigh threshold, as the stationary convection criterion remains independent of the modified particle-density increment parameters N_{B1} and N_{B2}. Furthermore, the thermo-nanofluid Lewis numbers emerge as key controlling parameters, significantly influencing the destabilization of stationary convection under top-heavy nanoparticle distributions. The presence of porosity is shown to further reduce the critical Rayleigh number, confirming the destabilizing role of porous resistance in hybrid nanofluid systems.

In addition, the study establishes explicit parametric conditions governing oscillatory behavior: overstability is suppressed when $N_{A_1} \leq 1$ and $N_{A_2} \leq 1$, while sufficient criteria for the nonexistence of oscillatory convection are obtained for $N_{A_1} > 1$ and $N_{A_2} > 1$. These results provide new quantitative insights into convection thresholds in hybrid nanofluid porous configurations, with potential relevance to thermal management, porous energy systems, and advanced heat transfer technologies involving multicomponent nanofluids.

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IZVOD

POČETAK KONVEKCIJE U POROZNOM SLOJU HIBRIDNOG NANOFLUIDA

Termička nestabilnost u nanofluidno-poroznim sistemima je od značajnog interesa zbog svoje direktne relevantnosti u naprednim tehnologijama upravljanja toplotom, skladištenja energije i poboljšanja prenosa toplote. Iako postoje opsežne studije o konvekciji u konvencionalnim nanofluidima, karakteristike stabilnosti hibridnih nanofluida zasićenih u poroznim medijumima - posebno pod kombinovanim efektima rastvorenih materija i nanočestica – ostaju nedovoljno istražene u literaturi. Cilj ove studije je da istraži početak termičke konvekcije u hibridnom sloju nano fluida zagrejanom odozdo, uzimajući u obzir uticaj zasićene porozne sredine. Koristi se linearni okvir stabilnosti, a upravljačke jednačine su analizirane korišćenjem tehnike normalnog režima kako bi se odredili uslovi za stabilnost i nestabilnost. Rezultati pokazuju da ugradnja dva različita tipa nanočestica u bazni fluid značajno odlaže stacionarnu konvekciju u poređenju sa suspenzijama sa jednom nanočesticom, čime se poboljšava termička stabilnost pod raspodelom teških čestica na vrhu. Šta više, utvrđeno je da prisustvo gradijenata rastvorenih materija podstiče raniji početak konvekcije u zagrejanom sloju. Ova otkrića pružaju nove uvide u kontrolu konvekcije u hibridnim nanofluidno-poroznim sistemima za termičke primene sledeće generacije.

Ključne reči: *Hibridni nanofluidni sloj; Braunovo kretanje; porozna sredina; stacionarna konvekcija; oscilatorna konvekcija*

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